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Corrigendum

Corrigendum to “Graphs and digraphs with all 2-factors isomorphic” [J. Combin. Theory Ser. B 92 (2) (2004) 395–404]

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ABSTRACT

We point out several errors in our article [M. Abreu, R.E.L. Aldred, M. Funk, B. Jackson, D. Labbate, J. Sheehan, Graphs and digraphs with all 2-factor isomorphic, J. Combin. Theory Ser. B 92 (2004) 395–404] which were caused by our misquoting of a theorem of C. Thomassen. We also describe how the correct statement of Thomassen's theorem, together with another of his theorems, can be used to obtain weaker results than those incorrectly stated in our original article.

We refer the reader to our original article [1] for all definitions and notation. The following necessary condition for a bipartite graph to be det-extremal is a special case of a result of Thomassen [4, Theorem 5.4].

Theorem 1. *Suppose G is a det-extremal bipartite graph and every edge of G belongs to a 1-factor. Then G has a vertex of degree at most three.*

We stated this result incorrectly in our original paper [1, Theorem 2.2] by leaving out the hypothesis that every edge of G belongs to a 1-factor of G. This led to several errors in [1]. These errors can be partially corrected by using another result of Thomassen which gives an upper bound on the

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minimum degree of all det-extremal bipartite graphs. His theorem [3, Theorem 3.2] is stated in terms of directed graphs and readily implies:

Theorem 2. *Suppose D is a digraph with n vertices and with no directed cycles of even length. Then D has a vertex of out-degree at most $\lfloor \log_2 n \rfloor$.*

Corollary 3. *Suppose G is a det-extremal bipartite graph with bipartition (A, B) where $|A| = |B| = n$. Then A contains a vertex of degree at most $\lfloor \log_2 n \rfloor + 1$.*

Proof. Suppose the corollary is false and let G be a counterexample. Then G is a det-extremal bipartite graph and each vertex in A has degree at least $\lfloor \log_2 n \rfloor + 2$. Choose a 1-factor $F = \{x_1 y_1, x_2 y_2, \ldots, x_n y_n\}$ of G and let D be the digraph obtained by directing all edges of G from A to B and then contracting each edge $x_i y_i \in F$ to a single vertex v_i. Each vertex of D has out-degree at least $\lfloor \log_2 n \rfloor + 1$ so, by Theorem 2, D has a directed cycle C of even length. Relabeling if necessary we may suppose that $C = v_1 v_2 \ldots v_{2m} v_1$. Let $M_1 = \{x_1 y_1, x_2 y_2, \ldots, x_{2m} y_{2m}\}$, $M_2 = \{x_1 y_2, x_2 y_3, \ldots, x_{2m} y_1\}$ and $F' = (F - M_1) \cup M_2$. Then $F \cup F'$ contains a unique circuit C', where $E(C') = M_1 \cup M_2$. Since C' has length congruent to zero modulo four, [1, Lemma 2.1] implies that $\mathrm{sgn}(F) \neq \mathrm{sgn}(F')$. This contradicts the fact that G is det-extremal.[1] □

Thomassen's result [3, Theorem 3.2] also implies that there exist det-extremal bipartite graphs on $2n$ vertices in which each vertex has degree at least $\lfloor \frac{1}{2} \log_2 n \rfloor$. Thus [1, Theorem 2.2] is false. We list below corrected statements of the results in [1] whose proofs relied on [1, Theorem 2.2]. We also indicate how our proofs in [1] can be modified to give the stated results.

Result 1. (Cf. [1, Corollary 2.3].) Let G be a 2-factor isomorphic bipartite graph with $2n$ vertices. Then each set in the bipartition of G contains a vertex of degree at most $\lfloor \log_2 n \rfloor + 2$.

Proof. The proof is similar to that of [1, Corollary 2.3] but uses Corollary 3 instead of [1, Theorem 2.2]. □

Result 2. (Cf. [1, Theorem 3.1].) Let D be a digraph with n vertices and X be a directed 2-factor of D. Suppose that either

(a) $d^+(v) \geqslant \lfloor \log_2 n \rfloor + 2$ for all $v \in V(D)$, or
(b) $d^+(v) = d^-(v) \geqslant 4$ for all $v \in V(D)$.

Then D has a directed 2-factor Y with $Y \ncong X$.

Proof. The proof is similar to that of [1, Theorem 3.1] but uses Corollary 3 to prove (a) and Theorem 1 to prove (b), instead of using [1, Theorem 2.2]. □

Result 3. (Cf. [1, Theorem 4.1 and Corollary 4.2].) Let G be a graph with n vertices and X be a 2-factor of G. Suppose that either

(a) $d(v) \geqslant 2(\lfloor \log_2 n \rfloor + 2)$ for all $v \in V(G)$, or
(b) G is a $2k$-regular graph for some $k \geqslant 4$.

Then G has a 2-factor Y with $Y \ncong X$.

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[1] The correspondence between det-extremal bipartite graphs and digraphs without directed cycles of even length is well documented, see for example [2, Section 2] or [4, Section 5]. We include a proof of Corollary 3 for the sake of completeness.

Proof. The proof of (a) is similar to that of [1, Theorem 4.1] but uses Result 2(a) instead of [1, Theorem 3.1]. Part (b) follows easily from Result 2(b). □

We do not know whether Corollary 2.3, Theorem 3.1, Theorem 4.1 and Corollary 4.2 of [1] are true or false. In particular the following problems are open.

Problem 4. Do there exist 2-factor isomorphic bipartite graphs of arbitrarily large minimum degree?

Problem 5. Do there exist 2-factor isomorphic regular graphs of arbitrarily large degree?

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